with $t$ appearing explicitly in the expression on the R.H.S. (right-hand side) of the equation, we must treat $t$ as a quantity to be scaled as a voltage. Wherever $t$ appears on the R.H.S. replace it by $z$, and introduce a second equation

$$z = t \text{ or } \frac{dz}{dt} = 1 \tag{30}$$

$z$ has to be scaled, and this means that we have an upper boundary $t_{max}$ which, once chosen, cannot be exceeded. $t_{max}$ is the number of units of $t$ over which the solution is needed.

If $t_{max} = 50$, we have $z \leqq 50$ and introduce the bracket $[\frac{z}{5}]$.

This has nothing to do with the time scaling $\tau = \beta t$. It is the voltage scaling of the variable $z$. The choice of $\beta$ determines how fast the solution will reach $t_{max}$. Including time scaling the voltage representing $z$ is generated by solving the equation

$$\frac{d(\frac{z}{5})}{d\tau} = \frac{d(\frac{t}{5})}{d\tau} = \frac{1}{5\beta} = \frac{1}{50\beta}\,[10] \tag{31}$$

Where $\frac{1}{50\beta}$ is the coefficient of the potentiometer giving the input to the time base integrator. *(Figure 18)*

The voltage from this integrator is used whenever a term involving $z$ (originally $t$) is to be formed in the expression on the R.H.S. of the programmed equation.

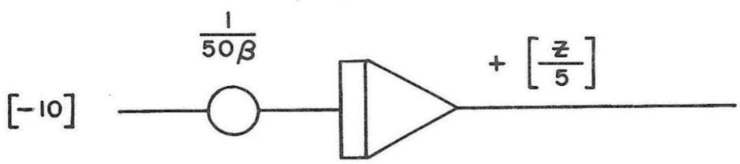


*Figure 26. Time Base Integrator.*

**ILLUSTRATIVE EXAMPLES.**

*1. A Simple Linear Problem.*

The dynamic behavior of the simple spring-mass-damping system as shown in *Figure 27* shall be investigated on the analog computer.

$x$ is the displacement of the mass measured from the position where the spring exerts a force mg to balance the gravitational force. The parameters $m$, $C$, and $K$ are constants. Initially the mass will be assumed to be released from rest at $x = X_0$. The displacement $x$, the velocity $x$ and the acceleration $x$ shall be recorded on a multichannel recorder.

System equation:

$$m\frac{d^2x}{dt^2} + C\frac{dx}{dt} + K_x = 0 \tag{32}$$

Rearranged equation:

$$\frac{d^2y}{dt^2} = -\frac{C}{m}\frac{dx}{dt} - \frac{K}{m}x \tag{33}$$

AMPLITUDE SCALING

In this problem an estimate of the magnitudes of the variables can be made by employing the undamped natural frequency of the system,